CHAPTER 1

Set Theory

Exercise

1. Let A = $\{x : x$ is a multiple of 3$\}$ and B = $\{x : x$ is multiple of 5$\}$. Then $A \cap B$ is given by
   (a) $\{3, 6, 9, ...\}$ (b) $\{5, 10, 15, 20, ...\}$
   (c) $\{15, 30, 45, ...\}$ (d) None of these
2. If A is the set of the divisors of the number 15, B is the set of prime numbers less than 10 and C is the set of even numbers less than 10, then $(A \cup C) \cap B$ is
   (a) $\{1, 3, 5\}$ (b) $\{1, 5, 7\}$
   (c) $\{2, 3, 5\}$ (d) $\{2, 4, 6\}$
3. If $A = \{1, 2, 3, 4\}$, $B = \{2, 4, 5, 6\}$ and $C = \{1, 2, 5, 7, 8\}$, then $(A \cup C) \cap B$ is equal to
   (a) $\{1, 2, 5\}$ (b) $\{2, 4, 5\}$
   (c) $\{1, 2, 4, 5, 7, 8\}$ (d) $\{1, 2, 3, 4, 5, 7, 8\}$
4. If $A = \{1, 2, 3, 4, ... 9\}$, $B = \{2, 4, 6, 7, 8\}$ and $C = \{3, 4, 5, 6, 9, 10\}$, then $(A – B) \cup C$ is equal to
   (a) $\{1, 3, 4, 5, 6, 9, 10\}$
   (b) $\{1, 2, 3, 4, 5, 6, 7, 8, 9\}$
   (c) $\{2, 4, 6, 7, 8\}$
   (d) $\{1, 3, 4, 5, 8, 9\}$
5. Let A and B be two sets.
   $(A \cup B)' \cup (A' \cap B)$ is equal to
   (a) A' (b) A
   (c) B' (d) None of these
6. Let $A = \{1, 2, 3\}$, $B = \{3, 4\}$ and $C = \{4, 5, 6\}$, then $[A \cup (B \cap C)]$ is equal to
   (a) $\{3\}$ (b) $\{1, 2, 3, 4\}$
   (c) $\{1, 2, 5, 6\}$ (d) $\{1, 2, 3, 4, 5, 6\}$
7. If A and B are two sets, then $A \cap (A \cup B)'$ equals to
   (a) A (b) B
   (c) ϕ (d) None of these
8. If $A = \{1, 3, 5, 7, 9, 11, 13, 15, 17\}$, $B = \{2, 4, ..., 18\}$ and N is the universal set, then $A' \cup \{[A \cup B] \cap B'\}$ is equal to
   (a) A (b) N
   (c) B (d) None of these
9. If sets A and B are defined as
   $A = \{(x, y) : y = e^x, x \in R\}$
   $B = \{(x, y): y = x, x \in R\}$, then
   (a) $B \subset A$ (b) $A \subset B$
   (c) $A \cap B = \phi$ (d) $A \cup B = A$
10. Let $A = \{(x, y) : y = e^x, x \in R\}$,
    $B = \{(x, y) : y = e^{-x}, x \in R\}$. Then
    (a) $A \cap B = \phi$ (b) $A \cap B \neq \phi$
    (c) $A \cup B = R^2$ (d) None of these
11. If sets A and B are defined as
    $$A = \{(x, y) : y = \frac{1}{x}, 0 \neq x \in R\}$$
    $B = \{(x, y) : y = -x, x \in R\}$, then
    (a) $A \cap B = A$ (b) $A \cap B = B$
    (c) $A \cap B = \phi$ (d) None
12. Which of the following is a singleton set ?
    (a) $\{x : x \in R, x^2 = x\}$
    (b) $\{x : x \in N, 3x = 4\}$
    (c) $\{x : x \in R, x^2 – 1 = 0\}$
    (d) $\{x : x$ is an integer which is neither + nor –$\}$
13. Which of the following statements is true ?
    (a) $\{a\} \subseteq \{\{a\}, b, c\}$ (b) $\{a\} \in \{\{a\}, b, c\}$
    (c) $\{a, b\} \subseteq \{\{a\}, b, c\}$ (d) $\{a, b\} \in \{\{a\}, b, c\}$
14. Consider the following statements
    I. Every subset of an infinite set is infinite.
    II. Every set has a proper subset.
    III. For any two sets A and B, either $A \subseteq B$ or $B \subseteq A$.
    IV. $\{x : x + 6 = 6\} = \phi$.
    Of these statements

(a) I, II and III are correct.
(b) Only II and III are correct.
(c) Only II and IV are correct.
(d) None is correct.

15. If X = $\{8^n - 7n - 1 : n \in N\}$ and Y = $\{49(n-1) : n \in N\}$, then
(a) $X \subset Y$ (b) $Y \subset X$
(c) $X = Y$ (d) None of these

16. If A is the set of all positive integers and B is the set of all negative integers, then $A \cup B$ is
(a) the set of all integers.
(b) the set of all integers excluding 0.
(c) the set of all non-negative integers.
(d) None of the above

17. Two finite sets are having m and n elements. The total number of subsets of first set is 56 more than the total number of subsets of the second set. The value of m and n are
(a) $m = 7, n = 6$ (b) $m = 6, n = 3$
(c) $m = 5, n = 1$ (d) $m = 8, n = 7$

18. If A and B are subsets of X, then
(a) $A \cup B = X \Rightarrow A' = B$
(b) $A \cup B = X \Rightarrow B' = A$
(c) $A \cup B = X \Rightarrow A \cap B = \phi$
(d) $A \cup B = X$ and $A' = B \Rightarrow A \cap B = \phi$

19. $A \cap B = \phi \Rightarrow$
(a) $A = \phi$ or $B = \phi$
(b) $A = \phi$ and $B = \phi$
(c) $A = \phi$ and $B \neq \phi$
(d) None of these

20. If A = $\{x \in N : 3 < x < 12\}$ and B = $\{x \in N : x$ is even, $x < 15)$, then
(a) $A \cup B = \{4, 6, 8, 10, 12, 14)$
(b) $A \cap B = \{4, 6, 8, 10\}$
(c) $A / B = \{5, 7, 9, 11, 13\}$
(d) $B / A = \{2, 8, 12, 14\}$

21. If A, B, C be three sets such that $A \cup B = A \cup C$ and $A \cap B = A \cap C$, then
(a) A = B (b) B = C
(c) A = C (d) A = B = C

22. In a group of 1500 persons, 1250 can speak English and 900 can speak Hindi. How many persons can speak Hindi only?
(a) 150 (b) 250
(c) 350 (d) 600

23. In a school, there are 20 teachers who can teach either Maths or Physics. There are 12 teachers who can teach Maths and there are 9 teachers who teach both Maths and Physics. The number of teachers who teach Physics is
(a) 13 (b) 17
(c) 19 (d) 23

24. From 50 students taking examination in Mathematics, Physics and Chemistry, 37 passed Mathematics, 24 Physics and 43 Chemistry. Atmost 19 passed Mathematics and Physics, atmost 29 passed Mathematics and Chemistry and atmost 20 passed Physics and Chemistry. The largest possible number of students that could have passed all three examinations is
(a) 11 (b) 12
(c) 13 (d) 14

25. An investigator interviewed 100 students to determine the performance of three drinks; milk, coffee and tea. The investigator reported that 10 students take all three drinks milk, coffee and tea; 20 students take milk and coffee; 25 students take milk and tea; 30 students take coffee and tea; 12 students take milk only; 5 students take coffee only and 8 students take tea only. Then the number of students who did not take any of three drinks is
(a) 10 (b) 20
(c) 25 (d) 30

26. In a class, 70 students wrote two tests viz, test–I and test–II. If 50% of the students failed in test–I and 40% of the students failed in test–II then how many students passed in both the tests ?
(a) 21 (b) 7
(c) 28 (d) 14

27. In a office, every employee likes at least one of tea, coffee and milk. The number of employees who like only tea, only coffee, only milk and all the three are all equal. The number of employees who like only tea and coffee, only coffee and milk and only tea and milk are equal and each is equal to the number of employees who like all the three. Then a possible value of the number of employees in the office is
(a) 65 (b) 90
(c) 77 (d) 85

28. There are 100 families in a society, 40 families by newspaper A, 30 families buy newspaper B, 30 families buy newspaper C, 10 families buy newspaper A and B, 8 families buy newspaper B and C, 5 families buy newspaper A and C, and 3 families by all the three newspapers then the number of families who do not buy any newspaper is
(a) 20 (b) 80
(c) 0 (d) 10

29. In a class of 55 students, the number of students studying different subjects are, 23 in Mathematics, 24 in Physics, 19 in Chemistry, 12 in Mathematics and Physics, 9 in Mathematics and Chemistry, 7 in Physics and Chemistry and 4 in all the three subjects.

The number of students who have taken exactly one subject is

(a) 6 (b) 9

(c) 7 (d) None of these

30. If $A = \{x \in R : x^2 + 6x - 7 < 0\}$ and $B = \{x \in R : x^2 + 9x + 14 > 0\}$, then which of the following is/are correct? **[NDA-II 2016]**

1. $A \cap B = \{x \in R : -2 < x < 1\}$
2. $A / B = \{x \in R : -7 < x < -2\}$

Select the correct answer using the code given below

(a) Only 1 (b) Only 2

(c) Both 1 and 2 (d) Neither 1 nor 2

ANSWERS

1.	(c)	**2.**	(c)	**3.**	(b)	**4.**	(a)	**5.**	(a)	**6.**	(b)	**7.**	(c)	**8.**	(b)	**9.**	(c)	**10.**	(b)
11.	(c)	**12.**	(d)	**13.**	(b)	**14.**	(d)	**15.**	(a)	**16.**	(b)	**17.**	(b)	**18.**	(d)	**19.**	(d)	**20.**	(b)
21.	(b)	**22.**	(b)	**23.**	(b)	**24.**	(d)	**25.**	(b)	**26.**	(b)	**27.**	(c)	**28.**	(a)	**29.**	(d)	**30.**	(a)

Explanations

1. (c) $A = \{3, 6, 9, 12, ...\}$
$B = \{5, 10, 15, 20, ... \}$
$A \cap B = \{15, 30, 45, ...\}$

2. (c) $A = \{1, 3, 5, 15\}, B = \{2, 3, 5, 7\}, C = \{2, 4, 6, 8\}$
$(A \cup C) \cap B$
$= \{1, 2, 3, 4, 5, 6, 8, 15\} \cap \{2, 3, 5, 7\}$
$= \{2, 3, 5\}$

3. (b) Given, $A = \{1, 2, 3, 4\}, B = \{2, 4, 5, 6\}$
and $C = \{1, 2, 5, 7, 8\}$
Then, $A \cup C = \{1, 2, 3, 4, 5, 7, 8\}$
and $(A \cup C) \cap B = \{2, 4, 5\}$

4. (a) $A = \{1, 2, 3, 4, ...9\}, B = \{2, 4, 6, 7, 8\}$
and $C = \{3, 4, 5, 6, 9, 10\}$
$A - B = \{1, 3, 5, 9\}$
then $(A - B) \cup C = \{1, 3, 4, 5, 6, 9, 10\}$

5. (a) $(A \cup B)' \cup (A' \cap B)$
$= (A' \cap B') \cup (A' \cap B)$
$= A' \cap (B' \cup B)$
$= A' \cap U = A'$

6. (b) $A = \{1, 2, 3\}, B = \{3, 4\}$ and $C = \{4, 5, 6\}$
$A \cup (B \cap C) = \{1, 2, 3\} \cup \{4\}$
$= \{1, 2, 3, 4\}$

7. (c) $A \cap (A \cup B)' = A \cap (A' \cap B')$
$= (A \cap A') \cap B$
$= \phi \cap B = \phi$

8. (b) $A' \cup \{[A \cup B] \cap B'\}$
Since, here $B \cap A = \phi$
$\Rightarrow B' = A$
Hence, $A' \cup \{[A \cup B] \cap A\}$
$= A' \cup A = N$

9. (c) $A = \{(x, y) : y = e^x, x \in R\}$
Put $x = -1, 0, 1$ ....
$\Rightarrow A = \{(0, 1), (1, e), (-1, e^{-1})...\}$
and $B = \{(x, y) : y = x, x \in R\}$
$\Rightarrow B = \{(0, 0), (1, 1), (-1, -1).....\}$
So, $A \cap B = \phi$

10. (b) $A = \{(x, y) : y = e^x, x \in R\}$
and $B = \{(x, y) : y = e^{-x}, x \in R\}$
On putting $x = -1, 0, 1, ...$
we get $A = \{(0, 1), (1, e), (-1, e^{-1}) ....\}$
and $B = \{(0, 1) (1, e^{-1}), (-1, e)....\}$
$\Rightarrow A \cap B = \{(0, 1)\}$
or $A \cap B \neq \phi$

11. (c) $\left\{(x, y) : y = \frac{1}{x}, 0 \neq x \in R\right\}$
and $B = \{(x, y) : y = -x, x \in R\}$
Put $x = ...-2, -1, 1, 2 ...$
$A = \{(-1,-1),\left(-2,-\frac{1}{2}\right)(1,1)\left(2,\frac{1}{2}\right)...\}$
$B = \{(-1, 1) (-2, 2) (1, -1) (2, -2) ...\}$
$\Rightarrow A \cap B = \phi$

12. (d) $\{x : x$ is an integer which is neither + nor – $\}$
$= \{0\}$ = singleton set.

13. (b) $\{a\} \in \{\{a\}, b, c\}\}$

14. (d) (I) Take a set of natural numbers N, which is an infinite set
and $A = \{1, 2, 3\} \subset N$.
Here, A is not an infinite set
So, (I) is not correct.
(II) Take an example of null set.
Number of elements = 0

Number of subsets = $2^{\circ} = 1$
which is ϕ itself. It is not a proper subset.
So, statement (II) is false.
(III) We cannot say $A \subseteq B$ or $B \subseteq A$
unless their elements are known.
(IV) $\{x : x + 6 = 6\} = \{\phi\} \neq \phi$
So no statement is correct.

15. (a) $X = \{8^n - 7n - 1 : n \in N\}$
and $Y = \{49(n - 1) : n \in N\}$
Put $n = 1, 2, 3, ...$
So, $X = \{0, 49, 490, ...\}$
and $Y = \{0, 49, 98, ...\}$
$\because$ Y includes all elements of X.
Hence, $X \subset Y$

16. (b) Given, $A = \{1, 2, 3, ...\}$
and $B = \{-1, -2, -3, ...\}$
So , $A \cup B = ...\{-3, -2, -1, 1, 2, 3...\}$
i.e., $A \cup B$ is the set of all integers excluding 0.

17. (b) Let two sets having m and n elements are A and B respectively.
Number of subsets of set A = 2^m
Number of subsets of set A = 2^n
Given, $2^m = 2^n + 56$
$\Rightarrow 2^m - 2^n = 56$
By Hit and Trial Method,
$m = 6$ and $n = 3$

18. (d) Given, $A \subset X$, $B \subset X$
and if $A \cup B = X$ and $A' = B$
$\Rightarrow A \cap B = \phi$ $\{\because A \cap A' = \phi\}$

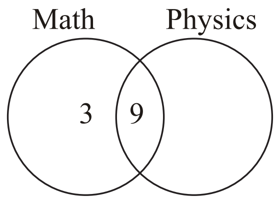
19. (d) $A \cap B = \phi$
Let $A = \{1, 2, 3\}$ and $B = \{4, 5, 6\}$
Here $A \neq \phi$ f and $B \neq \phi$
Still $A \cap B = \phi$
So, here no option is according to the given question.

20. (b) $A = \{x : x \in N$ and $3 < x < 12\}$
$A = \{4, 5, 6, 7, 8, 9, 10, 11\}$
$B = \{x \in N$ and x is even $< 15\}$
$B = \{2, 4, 6, 8, 10, 12, 14\}$
$A \cap B = \{4, 6, 8, 10\}$

21. (b) Given $A \cup B = A \cup C$ and $A \cap B = A \cap C$
This is possible only when B = C

22. (b) $n(E) = 1250$, $n(H) = 900$
$n(E \cup H) = 1500$, $n(H - E) = ?$
$n(E \cup H) = n(E) + n(H) - n(E \cap H)$
$1500 = 1250 + 900 - n(E \cap H)$
$n(E \cap H) = 2150 - 1500 = 650$
So only Hindi speaking ones
$n(H - E) = n(H) - n(E \cap H)$
$= 900 - 650 = 250$

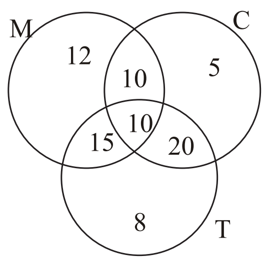
23. (b) Teachers teaching both Maths and Physics = 9
Maths teachers = 12
Teachers teaching only Maths = 12 – 9 = 3



Total teachers = 20
Teachers, teaching only physics
= 20 – (3 + 9) = 8
So, Physics teachers = 8 + 9 = 17

24. (d) $n(M \cup P \cup C) = 50$, $n(M) = 37$, $n(P) = 24$,
$n(C) = 43$, $n(M \cap P) = 19$, $n(M \cap C) = 29$,
$n(P \cap C) = 20$
$n(M \cup P \cup C) = n(M) + n(P) + n(C) - n(M \cap P)$
$- n(M \cap C) - n(P \cap C) + n(M \cap P \cap C)$
$50 = 37 + 24 + 43 - 19 - 29 - 20 + n(M \cap P \cap C)$
$\Rightarrow n(M \cap P \cap C) = 14$

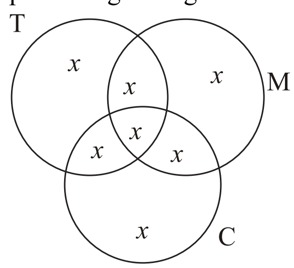
25. (b) Representing the given data in Venn diagram,



Total students = 100
So, number of students who did not take any of three drinks = 100 – (12 + 15 + 8 + 10 + 10 + 20 + 5)
= 20

26. (b) Number of students passed in Test I = 50%
Number of students passed in Test II = 60%
Number of students passed in both tests
= (50% + 60%) – 100% = 10% of 70 = 7

27. (c) Let the numbers of employees who like only tea $= x$
Then representing through Venn diagram



Total employees = $7x$
Hence, total employees can be only 77.

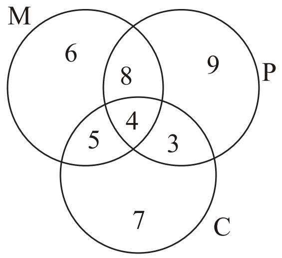
28. (a) $n(A) = 40$, $n(B) = 30$, $n(C) = 30$,
$n(A \cap B) = 10$, $n(B \cap C) = 8$, $n(C \cap A) = 5$,
and $n(A \cap B \cap C) = 3$

Then $n(A \cup B \cup C)$
$= n(A) + n(B) + n(C)$
$- n(A \cap B) - n(B \cap C) - n(C \cap A)$
$+ n(A \cap B \cap C)$
$= 40 + 30 + 30 - 10 - 8 - 5 + 3 = 80$

Hence, number of families who do not buy any newspaper $= 100 - 80 = 20$

29. (d) $n(M) = 23$, $n(M \cap P) = 12$, $n(P) = 24$, $n(M \cap C) = 9$, $n(C) = 19$, $n(P \cap C) = 7$ and $n(M \cap P \cap C) = 4$

Representing through Venn diagram



Number of students who have taken exactly one subject $= 7 + 9 + 6 = 21$

30. (a) $x^2 + 6x - 7 < 0$
$\Rightarrow (x + 7)(x - 1) < 0$
$\Rightarrow -7 < x < 1$
So, $A = \{x \in R : -7 < x < 1\}$
and $x^2 + 9x + 14 > 0$
$\Rightarrow (x + 2)(x + 7) > 0$
$\Rightarrow x < -7$ or $x > -2$
So, $B = \{x \in R : x < -7 \text{ or } x > -2\}$
Clearly, $A \cap B = \{x \in R : -2 < x < 1\}$
and $\frac{A}{B} = A - B = \{x \in R : -7 < x \le -2\}$
Hence, only statement 1 is correct.